Assignment 1

Sequence and Series; Mathematical Induction and the Binomial Theorem

Textbook Assignment: Chapters 1 and 2

In this course you will demonstrate that learning has taken place by correctly answering training items. The mere physical act of indicating a choice on an answer sheet is not in itself important; it is the mental achievement, in whatever form it may take, prior to the physical act that is important and toward which correspondence course learning objectives are directed. The selection of the correct choice for a correspondence course training item indicates that you have fulfilled, at least in part, the stated objective(s).

The accomplishment of certain objectives, for example, a physical act such as drafting a memo, cannot readily be determined by means of objective type correspondence course items; however, you can demonstrate by means of answers to training items that you have acquired the requisite knowledge to perform the physical act. The accomplishment of certain other learning objectives, for example, the mental acts of comparing, recognizing, evaluating, choosing, selecting, etc., may be readily demonstrated in a correspondence course by indicating the correct answers to training items.

The comprehensive objective for this course has already been given. It states the purpose of the course in terms of what you will be able to do as you complete the course.

The detailed objectives in each assignment state what you should accomplish as you progress through the course. They may appear singly or in clusters of closely related objectives, as appropriate; they are followed by items which will enable you to indicate your accomplishment.

All objectives in this course are learning objectives and items are teaching items. They point out important things, they assist in learning, and they should enable you to do a better job for the Navy.

This self-study course is only one part of the total Navy training program; by its very nature it can take you only part of the way to a training goal. Practical experience, schools, selected reading, and the desire to accomplish are also necessary to round out a fully meaningful training program.

● Make the following changes in your textbook:

In lines 1 and 2 (left column), page 16, change: "is not defined" to read "the series is divergent"

In line 16 (left column), page 16, add: "Therefore the series is divergent" after "$\lim_{n \to \infty} S_n = \infty$"

In problem #1 (left column), page 16, change: "3^n" to read "3n"

In lines 4 and 7 (right column), page 16, change: "3^n" to read "$3 \cdot 2^{n-1}$"

Learning Objective:

Solve problems involving arithmetic and geometric sequences by applying appropriate formulas.

1-1. Write the 5th term of the arithmetic sequence $1, \frac{3}{2}, 2, \ldots$.

1. 3
2. $\frac{7}{2}$
3. 4
4. 5

1-2. What is the 22nd term of the arithmetic sequence $1, \frac{5}{3}, \frac{7}{3}, \ldots$?

1. 8
2. 11
3. 15
4. 22

1-3. How many terms are in a sequence if the last term is 49, the difference is 3, and the first term is -5?
1. 13
2. 16
3. 19
4. 24

1-4. What is the first term of the arithmetic sequence whose 4th term is 16 and 8th term is 40?
1. 4
2. 2
3. 0
4. -2

1-5. What would be the mean of the sequence whose first term is 32 and last term is 64?
1. 64
2. 48
3. 32
4. 28

1-6. If there are three means between -8 and +28 then one of the means is
1. 1
2. 5
3. 18
4. 21

1-7. The sum of the first 50 terms of the series 3, 8, 13, . . . equals
1. 4,750
2. 5,325
3. 5,575
4. 6,275

1-8. Find the sum of the sequence having $d = 4$, $\ell = 37$, and $n = 7$.
1. 275
2. 250
3. 200
4. 175

1-9. Find the fourth term of an arithmetic sequence using the following information:
$n = 9 \quad \ell = 30 \quad S_n = 81$

1. $4\frac{1}{4}$
2. $3\frac{3}{4}$
3. $2\frac{1}{2}$
4. $1\frac{3}{4}$

1-10. What is the common ratio (r) of the sequence

$$1, \frac{1}{5}, \frac{1}{25}, \frac{1}{125}, \ldots ?$$

1. $-\frac{4}{5}$
2. $\frac{1}{25}$
3. $\frac{1}{5}$
4. $\frac{4}{5}$

1-11. Find the last term of the sequence where $a = 2$, $n = 7$, and $r = 4$.
1. 64
2. 1,950
3. 6,254
4. 8,192

1-12. What is the first term of a geometric sequence whose sixth term is 24 and seventh term is 8?
1. 3,484
2. 5,832
3. 6,446
4. 8,292

1-13. Find the one geometric mean between $\frac{1}{3}$ and $\frac{4}{27}$.

1. $\pm\frac{1}{6}$
2. $\pm\frac{2}{9}$
3. $\pm\frac{2}{11}$
4. $\pm\frac{2}{15}$

1-14. What is the sum of the first five terms in a geometric series whose first term is 1 and whose common ratio is 4?
1. 341
2. 322
3. 302
4. 285

1-15. What is the sum of a geometric series if $a = 2$, $r = \frac{1}{3}$, and $\ell = \frac{2}{243}$?

1. $\frac{852}{243}$
2. $\frac{728}{243}$
3. $\frac{649}{243}$
4. $\frac{528}{243}$

Learning Objective:

Derive formulas used to solve for various unknown elements in an infinite series, and determine some specified infinite-series terms.

1-16. The term given to a series that continues indefinitely is
   1. a finite series
   2. an infinite series
   3. a geometric series
   4. an arithmetic series

1-17. The sum of the infinite series $6 + \frac{9}{2} + \frac{27}{8} + \cdots$ is
   1. 16
   2. 18
   3. 20
   4. 24

1-18. What is the third term of the series whose n^{th} term is $\frac{6n - 4}{2n + 1}$?
   1. 4
   2. 3
   3. 2
   4. 1

1-19. What formula would represent the n^{th} term for the series $\frac{1}{4} + \frac{1}{8} + \frac{1}{12} + \cdots$?
   1. $\frac{1}{5n}$
   2. $\frac{1}{4n}$
   3. $\frac{1}{3n}$
   4. $\frac{1}{2n}$

1-20. What is the n^{th} term for the series $2 + \frac{3}{4} + \frac{4}{9} + \frac{5}{16} + \cdots$?
   1. $\frac{3n + 1}{2n}$
   2. $\frac{2n + 2}{n^2}$
   3. $\frac{n + 1}{n^2}$
   4. $\frac{2n}{n}$

Learning Objective:

Apply various tests to determine if given infinite series' are convergent or divergent, and why, and recognize the characteristics of a harmonic series.

● Items 1-21 through 1-38 relate to convergence and divergence. $U_n = n^{th}$ term.

1-21. The series $\frac{1}{10} + \frac{1}{20} + \frac{1}{40} + \cdots$ is
   1. convergent because $\lim_{n \to \infty} S_n = \frac{1}{5}$
   2. divergent because $\lim_{n \to \infty} U_n \neq 0$
   3. divergent because $\lim_{n \to \infty} U_n$ does not exist
   4. convergent or divergent because $\lim_{n \to \infty} S_n = \infty$

1-22. The series $2 + 8 + 32 + \cdots$ is
   1. divergent because $\lim_{n \to \infty} S_n \neq 0$
   2. divergent because $\lim_{n \to \infty} S_n = \infty$
   3. convergent because $\lim_{n \to \infty} S_n = \infty$
   4. convergent because $\lim_{n \to \infty} U_n$ does not exist

1-23. The series $-1 + 3 + 7 + \cdots$ is convergent.

1-24. If U_n is the n^{th} term of a series, $\lim_{n \to \infty} U_n = 0$ represents a
   1. proof of convergence
   2. proof of divergence
   3. necessary condition for convergence
   4. necessary condition for divergence

1-25. What can be said about the convergence or divergence of a series for which $U_n = 1 - \frac{1}{n + 1}$?
   1. Convergent because $\lim_{n \to \infty} U_n = 0$
   2. Convergent because $\lim_{n \to \infty} U_n = 1$
   3. Divergent because $\lim_{n \to \infty} U_n \neq 0$
   4. Convergent or divergent because $\lim_{n \to \infty} U_n \neq 0$

1-26. The series
$\frac{3}{4} + \frac{4}{5} + \frac{5}{6} + \cdots + \frac{n+2}{n+3} + \cdots$ is

1. divergent because $\lim_{n \to \infty} U_n \neq 0$
2. divergent because $\lim_{n \to \infty} U_n = \infty$
3. convergent because $\lim_{n \to \infty} U_n = 1$
4. divergent or convergent because $\lim_{n \to \infty} U_n = 0$

● Before taking the limit, divide the numerator and denominator by the highest power of n. A preliminary test for divergence which may save considerable time is to examine the n^{th} term of a series as n approaches infinity by the method given in your text.

1-27. This preliminary test for divergence shows that the series whose n^{th} term is $\frac{2n}{n^2+1}$ is

1. convergent because $\lim_{n \to \infty} \frac{2n}{n^2+1} = 0$
2. divergent because $\lim_{n \to \infty} \frac{2n}{n^2+1} = 1$
3. divergent because $\lim_{n \to \infty} \frac{2n}{n^2+1} = \infty$
4. convergent or divergent because $\lim_{n \to \infty} \frac{2n}{n^2+1} = 0$

1-28. What two conditions must be met for a geometric series to be convergent?

1. $\lim_{n \to \infty} U_n = 0$ and $|r| < 1$
2. $\lim_{n \to \infty} U_n = 0$ and $|r| > 1$
3. $\lim_{n \to \infty} U_n \neq 0$ and $|r| < 1$
4. $\lim_{n \to \infty} U_n \neq 0$ and $|r| > 1$

1-29. Which of the following is not a characteristic of a harmonic series?

1. $\lim_{n \to \infty} U_n = 0$
2. $\lim_{n \to \infty} S_n = \infty$
3. It is always convergent.
4. The reciprocals of the terms form an arithmetic series.

1-30. The series $a_1 + a_2 + \cdots + a_n$ is known to be convergent, and x_n of the series, $x_1 + x_2 + \cdots + x_n$ is compared to a_n for all corresponding terms. The x series is known to be

1. convergent if $x_n \geq a_n$
2. convergent if $x_n \leq a_n$
3. divergent if $x_n \geq a_n$
4. divergent if $x_n \leq a_n$

● Equations (1) through (4) on page 17 of the text show the "reference" or "t" series. For each of the following unknown or "U" series in items 1-31 through 1-33, select an appropriate "t" series. By the comparison test, determine if the "U" series is convergent or divergent.

1-31. The series $\frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \cdots$ is

1. divergent because $U_n \geq \frac{1}{n}$ of t-series
2. convergent because $U_n \leq \frac{1}{2^n}$ of t-series
3. convergent because $U_n \geq ar^n$ of t-series
4. convergent because $U_n \leq \frac{1}{n}$ of t-series

1-32. The series $\frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \cdots$ is

1. convergent because $U_n \leq \frac{1}{n^p}$ of t-series
2. convergent because $U_n \leq \frac{1}{2n}$ of t-series
3. convergent because $U_n < \left(\frac{1}{3}\right)^{n-1}$ of t-series
4. divergent because $U_n \geq \frac{1}{n}$ of t-series

1-33. The series $2 + 1 + \frac{2}{3} + \frac{1}{2} + \cdots + \frac{2}{n}$ is

1. divergent because $U_n < \frac{1}{n}$ of the t-series
2. divergent because $U_n > \frac{1}{n}$ of the t-series
3. convergent because $U_n < \frac{1}{2^n}$ of the t-series
4. convergent because $U_n > \frac{1}{2^n}$ of the t-series

1-34. Which statement is true concerning the convergence of the infinite series $U_1 + U_2 + U_3 + \cdots$, if $\lim_{n \to \infty} \left| \frac{U_{n+1}}{U_n} \right| = 0$,

where $U_1, U_2, U_3 \ldots$ are positive?

1. It is convergent.
2. It is convergent only for n=3.
3. It is not convergent.
4. It may or may not be convergent.

● Use the ratio test to investigate the convergence of the indicated series in items 1-35 through 1-38.

1-35. $\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots$

1. $\lim\limits_{n \to \infty} \frac{2^n}{2^n + 1} = 2$; diverges

2. $\lim\limits_{n \to \infty} \frac{2^n}{2^{n+1}} = \infty$; diverges

3. $\lim\limits_{n \to \infty} \frac{2^n}{2^{n+1}} = \frac{1}{2}$; converges

4. $\lim\limits_{n \to \infty} \frac{2^n}{2^{n+1}} = 1$; test fails

1-36. $\frac{3}{1 \cdot 2} + \frac{3^2}{2 \cdot 3} + \frac{3^3}{3 \cdot 4} + \cdots + \frac{3^n}{n(n+1)} + \cdots$

1. $\lim\limits_{n \to \infty} \frac{n+2}{3n} = \frac{1}{3}$; converges

2. $\lim\limits_{n \to \infty} \frac{3n}{n(n+1)} = 0$; converges

3. $\lim\limits_{n \to \infty} \frac{3n}{n+2} = 3$; diverges

4. $\lim\limits_{n \to \infty} \frac{n+2}{3n} = 1$; test fails

1-37. $\frac{3}{1} + \frac{3^2}{3} + \frac{3^3}{5} + \cdots + \frac{3^n}{2n-1} + \cdots$

1. $\lim\limits_{n \to \infty} \frac{3^n}{2n+1} = \frac{3}{2}$; diverges

2. $\lim\limits_{n \to \infty} \frac{6n-3}{2n+1} = 3$; diverges

3. $\lim\limits_{n \to \infty} \frac{6n-3}{2n+1} = \infty$; diverges

4. $\lim\limits_{n \to \infty} \frac{3n}{2n-1} = 0$; converges

1-38. $\frac{1}{2} + \frac{2!}{2^2} + \frac{3!}{2^3} + \cdots + \frac{n!}{2^n} + \cdots$

1. $\lim\limits_{n \to \infty} \frac{n+1}{2} = \infty$; diverges

2. $\lim\limits_{n \to \infty} \frac{2^{(n+1)}}{n!} = 2$; diverges

3. $\lim\limits_{n \to \infty} \frac{2}{n+1} = 0$; converges

4. $\lim\limits_{n \to \infty} \frac{n!}{2^n} = \frac{1}{2}$; converges

Learning Objective:

Recognize the theory of proof by mathematical induction and use this method of proof to verify a mathematical formula.

● In answering items 1-39 through 1-42, refer to the formula $\frac{1}{2} + 1 + \frac{3}{2} + \cdots + \frac{n}{2} = \frac{n(n+1)}{4}$.

1-39. To prove the formula true by mathematical induction as explained in the text, it must first be proven correct for n =
1. 0
2. 1/2
3. 1
4. K-1

1-40. Since the formula works for the case in item 1-39; assume the formula is correct for n = K, where K is any whole number. Therefore, equation (1)

$$\frac{1}{2} + 1 + \frac{3}{2} + \cdots + \frac{K}{2} = \frac{K(K+1)}{4}$$

is assumed to be true and we proceed to prove the formula correct for n =
1. 0
2. 2
3. K-1
4. K+1

1-41. For the value of n found in the preceding item, the formula can be written

1. $\frac{1}{2} + 1 + \frac{3}{2} + \cdots + \frac{7}{2} = \frac{K(K+1)}{4}$

2. $\frac{1}{2} + 1 + \frac{3}{2} + \cdots + \frac{K+1}{2} = \frac{K(K+1)}{4}$

3. $\frac{1}{2} + 1 + \frac{3}{2} + \cdots + \frac{7}{2} = \frac{K^2+3K+2}{4}$

4. $\frac{1}{2} + 1 + \frac{3}{2} + \cdots + \frac{K+1}{2} = \frac{K^2+3K+2}{4}$

1-42. Let the equation, obtained as the answer in the previous item, be equation (2). To prove the original formula for $n = K+1$, the assumed relation (1),

$$\frac{1}{2} + 1 + \frac{3}{2} + \cdot \cdot \cdot + \frac{K}{2} = \frac{K(K+1)}{4}$$

must be manipulated so that it becomes identical to equation (2). Since the left sides of (1) and (2) differ by

$\frac{K+1}{2}$ we add $\frac{K+1}{2}$ to each side of (1),

that is, $\frac{1}{2} + 1 + \frac{3}{2} + \cdot \cdot \cdot + \frac{K}{2} + \frac{K+1}{2}$

$$= \frac{K(K+1)}{4} + \frac{K+1}{2} \left(\text{equation (3)}\right).$$

Which of the following conclusions may be drawn concerning equations (2) and (3)

1. (2) and (3) say the same thing and the original formula is untrue.
2. (2) and (3) say the same thing and the original formula is true.
3. (2) and (3) say different things and the original formula is untrue.
4. (2) and (3) say different things and the original formula is true.

Learning Objective:

Recognize the characteristics of an expanded binomial and solve or evaluate problems based upon the expansion of a binomial.

1-43. How many terms are in the expansion of $(x + y)^7$?

1. 9
2. 8
3. 7
4. 6

1-44. The sum of the exponents of each term in the expansion of $(x + y)^8$ is

1. 9
2. 8
3. 7
4. 6

1-45. The coefficient of the fourth term of the expansion of $(x + y)^7$ is

1. 15
2. 20
3. 25
4. 35

1-46. The third term of the expansion of $[(3x) + (-2y)]^3$ is

1. $36xy^2$
2. $18xy^2$
3. $12xy^2$
4. $3xy^2$

1-47. Evaluate $(1 + 0.02)^6$ to the nearest hundredth.

1. 1.13
2. 1.14
3. 1.15
4. 1.16

1-48. Evaluate $(1 + 0.04)^4$ to the nearest hundredth.

1. 1.15
2. 1.16
3. 1.17
4. 1.18

1-49. Evaluate $(0.96)^4$ to the nearest hundredth by calculating the value of the appropriate binomial expansion.

1. 0.70
2. 0.80
3. 0.85
4. 0.99

1-50. What is the third term of $(m^3 - 2)^5$?

1. $10m^6$
2. $20m^6$
3. $30m^3$
4. $40m^9$

1-51. What is the sixth term of $(x + y)^{10}$?

1. $252x^5y^5$
2. $252x^5y^6$
3. $252x^6y^5$
4. $252x^6y^6$

1-52. The third term of $(3x + 3y)^5$ is

1. $90x^3y^2$
2. $180x^3y^2$
3. $270x^3y^2$
4. $2430x^3y^2$

1-53. What is the sixth term of $(2m^3 - x^2)^8$?

1. $56m^6x^{10}$
2. $112m^9x^{10}$
3. $-224m^6x^{10}$
4. $-448m^9x^{10}$

1-54. What is the fourth term of the expansion of $(x + y)^{-3}$? (y is less than x.)

1. $-10x^{-6}y^3$
2. $-15x^{-6}y^3$
3. $20x^{-6}y^3$
4. $30x^{-6}y^3$

1-55. The fifth term of $(x + y)^{1/2}$ after expansion and simplification is

1. $-\frac{y^4}{64x^{7/2}}$

2. $-\frac{5y^4}{128x^{7/2}}$

3. $\frac{y^4}{32x^{7/2}}$

4. $\frac{5y^4}{32x^{7/2}}$

1-56. Evaluate $\sqrt[4]{12}$ to the nearest hundredth.

1. 1.86
2. 1.87
3. 1.88
4. 1.89

1-57. What number would be between 28 and 56 in the next row of a Pascal's triangle?

1. 69
2. 78
3. 81
4. 84

1-58. Using Pascal's triangle shown on page 28 of the text determine the coefficient of the fourth term of $(x + y)^n$ when $n = 8$.

1. 62
2. 56
3. 48
4. 42